Cairo University - Faculty of Science  
Department of Mathematics  
Final Exam of Calculus (I) (code: PTM 101)  
Petroleum Geosciences Program  

4 January 2011  
Time allowed: Two hours  
Total degree: 60 points

**Answer the following questions:**

**Question 1** (7+7+6 = 20 points)

(a) Find the solution set of the inequality $|5 - x| < 4$ and deduce the domain of the function $f(x) = \sqrt{|5 - x| - 4}$.

(b) If $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1+x}{x}$, find $(gof)(x)$ and its domain and range. Deduce $f^{-1}(x)$.

(c) Discuss the continuity of the function $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ in the domain $\mathbb{R}$.

**Question 2** (8+7+5 = 20 points)

(a) Find the two limits

1. $\lim_{x \to \infty} 2x \sin \left(\frac{1}{x}\right) + \frac{\cos x}{x}$

2. $\lim_{x \to 0} \frac{\tan^2(3x)}{1 - \sqrt{1+x^2}}$

(b) If $y = 1 + u^8$, $u = \frac{1-x}{1+x}$, then by using the chain rule find $\frac{dy}{dx}(0)$.

(c) Find the equation of the tangent line to the curve of the function $f(x) = \frac{x}{x+2}$ when $x = -4$.

**Question 3** (9+5+6 = 20 points)

(a) Find $\frac{dy}{dx}$ for the following functions:

1. $y = x^{\sin x} + 3^x$

2. $x^2 + xy + y^3 = 1$

3. $y = \tan^5 x + \cos^{-1}(\sin x^3)$

(b) By using L'Hospital's rule, find $\lim_{x \to 0} \frac{\ln(\cos(3x))}{\ln(\cos(5x))}$.

(c) Determine the local maximum and local minimum values of the function $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x^2 + 2x + 1$. 